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A novel Dynamic Fuzzy Sets Method Applied to Practical Teaching Assessment on Statistical Software

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Abstract

In this paper, we present a novel dynamic fuzzy sets (DFS) method, which is the generalization of fuzzy sets (FS) and the dynamization of intuitionistic fuzzy sets (IFS). First, by analyzing the degree of hesitancy, we propose a DFS model from IFS. Second, a multiple attribute decision making example applied to practical teaching assessment is given to demonstrate the application of DFS, and the simulation results show that the DFS method is more effective than the IFS method and the FS method. Finally, a multiple-level practical teaching assessment model is proposed according to DFS.

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1. Main text

Professor L. A. Zadeh's paper on fuzzy sets (FS, 1965) has influenced many researchers and has been applied to many application fields, such as pattern recognition, fuzzy reasoning, decision making, etc. In 1986, K. T. Atanassov introduced membership function, non-membership function and hesitancy function, and proposed the concept of intuitionistic fuzzy sets (IFS), which generalized the FS theory. In the research field of IFS, Yager discussed its characteristics (2009), and Xu (2007-2010), Wei (2009-2010), et al. applied it to

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decision making. Though many scholars studied IFS and applied it to decision making, most of their methods are suitable for static model and unsuitable for dynamic model. Considering that few references related to the study of dynamic decision-making from IFS was proposed, Xu (2008) presented a dynamic decision making model, which was also studied by Wei, Su, et al. (2009, 2011). However, traditional decision analysis models based on IFS do not involve the detachment of the absent party, which means that the decision-making results may be limited in the scope. Thus, in this paper, we present a novel DFS model by analyzing the hesitancy function.

First, we present the definition and the construction method of DFS. And then, we introduce some ranking functions of IFS and generalize it to DFS. Finally, we apply the DFS model along with its membership function to a multiple level practical teaching assessment problem. The simulation results show that the method introduced in this paper is more comprehensive and flexible than the IFS method and the FS method. Thus, this paper can provide valuable conclusion for the application research of FS to teaching assessment field, and the model of DFS is also useful for the generalization from fuzzy reasoning and intuitionistic fuzzy reasoning to dynamic fuzzy reasoning.

2. Construction of DFS

Definition 1. An IFS A in universe X is given by the following formula (Atanassov 1986):

$$A = \{ \langle x, u_A(x), v_A(x) \rangle \mid x \in X \}. \quad (1)$$

Where $u_A : X \rightarrow [0, 1]$, $v_A : X \rightarrow [0, 1]$ with the condition $0 \leq u_A(x) + v_A(x) \leq 1 \quad \forall x \in X$. The numbers $u_A(x) \in [0, 1]$, $v_A(x) \in [0, 1]$ denote a degree of membership and a degree of non-membership of x to A , respectively. For each IFS in X , we call $\pi_A(x) = 1 - u_A(x) - v_A(x)$ a degree of hesitancy of x to A , $0 \leq \pi_A(x) \leq 1$ for each $x \in X$.

Definition 2. A DFS A in universe X is denoted by:

$$A = \{ \langle x, \mu_A^*(x), v_A^*(x) \rangle \mid x \in X \}.$$

Where $\mu_A(x), v_A(x)$ and $\pi_A(x)$ are from definition 1, and we have $\mu_A^*(x) = \mu_A(x) + \lambda \pi_A(x)$ and $v_A^*(x) = v_A(x) + (1 - \lambda) \pi_A(x)$ with the condition $\lambda \in [0, 1]$. $\mu_A^*(x)$ and $v_A^*(x)$ are membership function and non-membership function of x to A , respectively.

Theorem 1. Let A be an DFS as mentioned above, then

$$\mu_A^*(x) + v_A^*(x) = 1. \quad (2)$$

According to definition 2, we have theorem 1.

From definition 2, let all sample data be divided into three parts, $\mu_A(x)$ being the firm support party of event A , $v_A(x)$ representing the firm opposition party of event A , and $\pi_A(x)$ showing all the absent party that may become either the support party or the opposition party. In the absent party, if there is $\lambda \pi_A(x)$ sample supporting event A and $(1 - \lambda) \pi_A(x)$ sample opposing event A , we have DFS denoted by definition 2.

Obviously, DFS is an extension of FS, and is a dynamization of IFS.

3. Ranking functions

Atanassov have introduced the following membership function, which is a kind of decision making method only using the membership degree (1986):

$$R_M(A) = \sum_{x \in X} w_A(x) \mu_A(x). \quad (3)$$

Similarity, we define a kind of decision making method only using the non-membership degree:

$$R_M(A) = \sum_{x \in X} w_A(x) v_A(x). \quad (4)$$

Considering the limitation of the membership function and the non-membership function, Chen and Tan proposed a dominant ranking function to make decisions (1994), which involves the membership degree and the non-membership degree:

$$R_D(A_i) = \sum_{x \in X} w_{A_i}(x) (\mu_{A_i}(x) - v_{A_i}(x)), i = 1, 2, \dots, n. \quad (5)$$

Considering the significance of the membership function, we have the following membership ranking function:

$$R_M(A^*) = \sum_{i=1}^n w_A(x_i) \mu_A^*(x_i) = \sum_{i=1}^n w_A(x_i) \mu_A(x_i) + \lambda \sum_{i=1}^n w_A(x_i) \pi_A(x_i). \quad (6)$$

Where $\lambda \in [0, 1]$.

From the dominant function, we define a weighted dominant ranking function of DFS as follows:

$$\begin{aligned} R_D(A^*) &= \sum_{i=1}^n w_A(x_i) (a \mu_A^*(x_i) - b v_A^*(x_i)) = \sum_{i=1}^n w_A(x_i) (a \mu_A^*(x_i) - b + b \mu_A^*(x_i)) \\ &= (a + b) \sum_{i=1}^n w_A(x_i) \mu_A^*(x_i) - b = \sum_{i=1}^n w_A(x_i) (a + b) [\mu_A(x_i) + \lambda \pi_A(x_i)] - b. \end{aligned} \quad (7)$$

Where $\lambda \in [0, 1]$, $a > 0$, and $b > 0$. From formula (7), the decision-making of the weighted dominant ranking function are similar to that of the membership function for DFS. In the following, we use the membership function of DFS to make decision.

4. Application to single attribute decision making

In the following, we will apply the DFS above to decision making. In order to show the difference among DFS, FS and IFS being applied to decision-making, we have example 1.

Example.1 Suppose that A is feasible alternatives set, $A = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9\}$. According to the following data from A_i , we will make a choice among A . Assume that A_i ($i=1,2,3,4,5,6,7,8,9$) are represented by IFS, shown as follows: $A_1 = \{ \langle x, 0.7, 0.3 \rangle \}$, $A_2 = \{ \langle x, 0.6, 0.3 \rangle \}$, $A_3 = \{ \langle x, 0.5, 0.3 \rangle \}$, $A_4 = \{ \langle x, 0.5, 0.4 \rangle \}$, $A_5 = \{ \langle x, 0.5, 0.5 \rangle \}$, $A_6 = \{ \langle x, 0.4, 0.5 \rangle \}$, $A_7 = \{ \langle x, 0.3, 0.5 \rangle \}$, $A_8 = \{ \langle x, 0.3, 0.6 \rangle \}$, $A_9 = \{ \langle x, 0.3, 0.7 \rangle \}$.

$0.3, 0.7>\}$, where $A_i = \{ \langle x, \mu_{A_i}(x), \nu_{A_i}(x) \rangle \mid x \in X \}$. For example, A_1 means that the degree of membership is $\mu_{A_1}(x) = 0.7$, and that the degree of non-membership is $\nu_{A_1}(x) = 0.3$, and then the degree of hesitancy is $\pi_{A_1}(x) = 0$ for $x \in X$. According to their practical significance, we have:

$$A_9 \subset A_8 \subset A_7 \subset A_6 \subset A_5 \subset A_4 \subset A_3 \subset A_2 \subset A_1.$$

In the following, we will compare the results calculated by the membership function of DFS with the results calculated by the membership function of IFS.

Using the membership function of IFS, we have $0.3 < 0.4 < 0.5 < 0.6 < 0.7$, and then the result will be:

$$A_9 = A_8 = A_7 \subset A_6 \subset A_5 = A_4 = A_3 \subset A_2 \subset A_1.$$

Using the non-membership function of IFS, we also have:

$$A_9 \subset A_8 \subset A_7 = A_6 \subset A_5 \subset A_4 \subset A_3 = A_2 = A_1.$$

Using the dominant function of IFS, we have:

$$A_9 \subset A_8 \subset A_7 \subset A_6 \subset A_5 \subset A_4 \subset A_3 \subset A_2 \subset A_1.$$

Using the membership function of DFS, we have Fig.1:

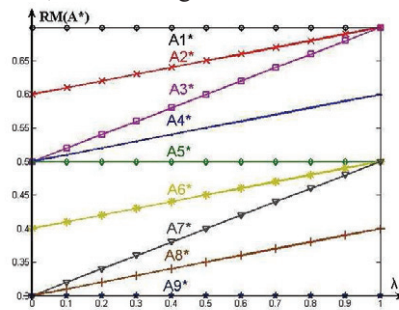


Fig. 1. Single attribute decision-making on DFS

For example, $R_M(A_2^*) = \sum_{i=1}^n w_{A_2}(x_i) \mu_{A_2}^*(x_i) = \mu_{A_2}^*(x_i) = \mu_{A_2}(x_i) + \lambda \pi_{A_2}(x_i) = 0.6 + 0.1\lambda$.

Similarity $R_M(A_k^*)(k=1,2,\dots,9)$ can be calculated.

Based on Fig.1, the result is $A_9 \subset A_8 \subset A_7 \subset A_6 \subset A_5 \subset A_4 \subset A_3 \subset A_2 \subset A_1$ for each $\lambda \in (0, 1)$. Thus, we conclude that the membership function of DFS match the original assumptions when $\lambda \neq 0$ and $\lambda \neq 1$.

5. Application to practical teaching assessment on statistical software

In the following, we will apply the membership function of DFS above to practical teaching assessment on statistical software according to dynamic fuzzy multiple attribute decision making. We illustrate the advantage of the DFS method by the following data from [5] (Xu, 2007). On account of the particularity and the specialty of the practical teaching on statistical software, we select three attributes to make decision: classroom exercises, homework assignments, statistical practice investigation report.

Example.2 A statistics instructor is planning to evaluate the effect of the most outstanding students who are studying statistical software. Five excellent students A_i ($i=1, 2, 3, 4, 5$) will be sorted. Suppose that three attributes C_1 (homework assignments), C_2 (practice investigation report), and C_3 (classroom exercises) are taken into consideration, the weight vector of the attributes C_j ($j=1,2,3$) is $w=(0.3,0.5,0.2)^T$. Assume that the characteristics of the options A_i ($i=1,2,3,4,5$) are represented by IFS, shown as follows:

$A_1 = \{ \langle C_1, 0.2, 0.4 \rangle, \langle C_2, 0.7, 0.1 \rangle, \langle C_3, 0.6, 0.3 \rangle \}$, $A_2 = \{ \langle C_1, 0.4, 0.2 \rangle, \langle C_2, 0.5, 0.2 \rangle, \langle C_3, 0.8, 0.1 \rangle \}$,
 $A_3 = \{ \langle C_1, 0.5, 0.4 \rangle, \langle C_2, 0.6, 0.2 \rangle, \langle C_3, 0.9, 0 \rangle \}$, $A_4 = \{ \langle C_1, 0.3, 0.5 \rangle, \langle C_2, 0.8, 0.1 \rangle, \langle C_3, 0.7, 0.2 \rangle \}$,
 $A_5 = \{ \langle C_1, 0.8, 0.2 \rangle, \langle C_2, 0.7, 0 \rangle, \langle C_3, 0.1, 0.6 \rangle \}$. Where the data show the excellent degree of them.

We will compare the results calculated by conventional distance measures of IFS ([5]) with the results calculated by the membership function of DFS derived from IFS.

From formulas (3), we obtain the results as follows:

$$R_M(A_1) = 0.3 \times 0.2 + 0.5 \times 0.7 + 0.2 \times 0.6 = 0.53, R_M(A_2) = 0.53, R_M(A_3) = 0.63, R_M(A_4) = 0.63, R_M(A_5) = 0.61.$$

From formulas (4), we obtain the results as follows:

$$R_{NM}(A_1) = 0.3 \times 0.4 + 0.5 \times 0.1 + 0.2 \times 0.3 = 0.23, R_{NM}(A_2) = 0.18, R_{NM}(A_3) = 0.22, R_{NM}(A_4) = 0.24, R_{NM}(A_5) = 0.18.$$

Since $R_M(A_3) = R_M(A_4) > R_M(A_5) > R_M(A_1) = R_M(A_2)$ and $R_{NM}(A_2) = R_{NM}(A_5) < R_{NM}(A_3) < R_{NM}(A_1) < R_{NM}(A_4)$, we get $A_5 \succ A_2 \succ A_1$ and $A_3 \succ A_4$. For example, from the membership degree $R_M(A_5) > R_M(A_1) = R_M(A_2)$ and the non-membership degree $R_{NM}(A_5) = R_{NM}(A_2) < R_{NM}(A_1)$, we obtain $A_5 \succ A_2 \succ A_1$. Hence, either A_5 or A_3 are the optimal decision-making.

From formulas (5), we obtain the results as follows:

$$R_D(A_1) = 0.3 \times (0.2 - 0.4) + 0.5 \times (0.7 - 0.1) + 0.2 \times (0.6 - 0.3) = 0.3, R_D(A_2) = 0.35, R_D(A_3) = 0.41, R_D(A_4) = 0.39, R_D(A_5) = 0.43.$$

According to the results in [5], we have the following results in Table 1.

Table 1. Assessment results based on ranking functions of IFS

Ranking function	A_1	A_2	A_3	A_4	A_5	Decision-making
<u>Xu1</u>	<u>0.4685</u>	<u>0.5083</u>	<u>0.555</u>	<u>0.5147</u>	<u>0.4954</u>	$A_3 \succ A_4 \succ A_2 \succ A_5 \succ A_1$
<u>Xu2</u>	<u>0.4813</u>	<u>0.5315</u>	<u>0.5793</u>	<u>0.5313</u>	<u>0.5130</u>	$A_3 \succ A_2 \succ A_4 \succ A_5 \succ A_1$
<u>Xu3</u>	<u>0.4679</u>	<u>0.4679</u>	<u>0.5583</u>	<u>0.551</u>	<u>0.5372</u>	$A_3 \succ A_4 \succ A_5 \succ A_2 = A_1$
Xu4	0.4591	0.4835	0.5589	0.5347	0.5331	$A_3 \succ A_4 \succ A_5 \succ A_2 \succ A_1$
<u>Membership function</u>	<u>0.53</u>	<u>0.53</u>	<u>0.63</u>	<u>0.63</u>	<u>0.61</u>	$A_3 = A_4 \succ A_5 \succ A_2 = A_1$
<u>Non-membership function</u>	<u>0.23</u>	<u>0.18</u>	<u>0.22</u>	<u>0.24</u>	<u>0.18</u>	$A_2 = A_5 \succ A_3 \succ A_1 \succ A_4$
Dominant function	0.3	0.35	0.41	0.39	0.43	$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$

From Table 1, we know that Xu4 method and the dominant ranking function satisfy $A_5 \succ A_2 \succ A_1$ and $A_3 \succ A_4$. A_3 is the optimal decision for the membership function, which is the same as in [5]. However, A_5 will be the optimal decision when we make use of the non-membership function and the dominant function.

From formula (6), we have:

$$R_M(A_1^*) = \sum_{i=1}^n w_A(x_i) \mu_A^*(x_i) = \sum_{i=1}^n w_A(x_i) \mu_A(x_i) + \lambda \sum_{i=1}^n w_A(x_i) \pi_A(x_i) \\ = 0.3 \times 0.2 + 0.5 \times 0.7 + 0.2 \times 0.6 + \lambda(0.3 \times 0.4 + 0.5 \times 0.2 + 0.2 \times 0.1) = 0.53 + 0.24\lambda.$$

Similarity, we get:

$$R_M(A_2^*) = 0.53 + 0.29\lambda, R_M(A_3^*) = 0.63 + 0.15\lambda, R_M(A_4^*) = 0.63 + 0.13\lambda, R_M(A_5^*) = 0.61 + 0.21\lambda.$$

Then we have Fig.2 as follows. From Fig.2, the results will be in Table.2

From Table.2, all of the results are satisfying $A_5 \succ A_2 \succ A_1$ and $A_3 \succ A_4$. And we know that the optimal decision is A_3 when $\lambda < 0.3333$ and A_5 when $\lambda > 0.3333$. According to definition 3, λ indicates the proportion of the absent party being converted into the support party, and $1-\lambda$ indicates the proportion of the convertible absent party being converted into the opposition party. Thus, it implies that when the proportion of the absent

party being converted into the support party is less than 33.33%, then A_3 is the most excellent student; otherwise, A_5 is the most excellent one. On account of the definitions of A_5 and A_3 , $A_5 = \{ \langle C_1, 0.8, 0.2 \rangle, \langle C_2, 0.7, 0. \rangle, \langle C_3, 0.1, 0.6 \rangle \}$, and $A_3 = \{ \langle C_1, 0.5, 0.4 \rangle, \langle C_2, 0.6, 0.2 \rangle, \langle C_3, 0.9, 0 \rangle \}$. There are much difference between A_5 and A_3 . We have $A_5 > A_3$ for attribute C_1 and attribute C_2 while $A_5 < A_3$ for attribute C_3 , which means that A_5 is more excellent than A_3 in the homework assignments and in the practice investigation report though his performance in classroom exercises is terrible. Considering that the weight of homework assignments and the weight of practice investigation report are more than the weight of classroom exercises, A_5 should be the best student.

In [5], Xu applied four kinds of distance measures to make decisions, only Xu4 method satisfies the basic conditions. In this paper, we use the membership function, the non-membership function and the dominant ranking function of IFS to make decision, only the dominant function satisfies the basic conditions. However, when applying the membership function of DFS to make decision, we conclude that the membership function of DFS satisfies all the conditions for each $\lambda \in [0,1]$. Furthermore, by analyzing the variation of the indeterminacy degree, we reveal a potential optimal decision-making A_5 and the reason for A_5 . In this paper, we assume that λ is a constant for all the attributes. However, in practice the proportion of the absent party may be different for all the attributes. Therefore, researchers can set more parameters to deal with real problems.

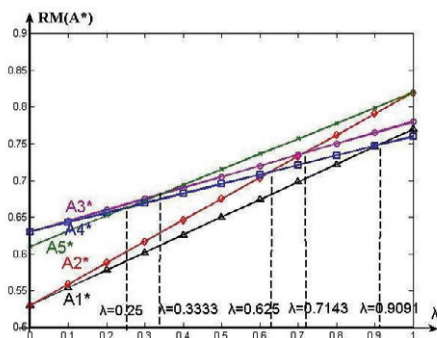


Fig. 2. Practical teaching assessment results on DFS

Table 2. Assessment results based on the membership function of DFS

Ranking function	Decision-making	Optimal decision-making
$0 < \lambda < 0.25$	$A_3 > A_4 > A_5 > A_2 > A_1$	A_3
$0.25 < \lambda < 0.3333$	$A_3 > A_5 > A_4 > A_2 > A_1$	A_3
$0.3333 < \lambda < 0.625$	$A_5 > A_3 > A_4 > A_2 > A_1$	A_5
$0.625 < \lambda < 0.7143$	$A_5 > A_3 > A_2 > A_4 > A_1$	A_5
$0.7143 < \lambda < 0.9091$	$A_5 > A_2 > A_3 > A_4 > A_1$	A_5
$0.9091 < \lambda < 1$	$A_5 > A_2 > A_3 > A_1 > A_4$	A_5

The experimental results above show that there is much difference between the multiple attribute decision making results of DFS and the results of IFS. The conventional IFS method is simple, but its decision making results are fixed as it is calculated by the conventional ranking functions. Thus, it is difficult to reveal the potential rules from all the available information when using the IFS method. Moreover, the results of the

DFS method are flexible, which can be adjusted to generating desirable results with the variation of the parameters. Furthermore, for supervised models, if the fixed decision making results of IFS are different from the results of practical data and the actual decision, the IFS method will not work. However, when using the DFS method, we can meet the needs of practical data and the actual decision by adjusting the parameters to appropriate values. All in all, the results above show that the DFS method is more effective than the IFS method.

6. Application to Multiple level practical teaching assessment model

For the example above, all the students are derived from an outstanding student set. Thus, we can assess them using the following formula (8).

$$R_M(A_k^*) = W \circ A_k^* = (w_{A_k}(x_1) \quad w_{A_k}(x_2) \quad \cdots \quad w_{A_k}(x_n)) \circ \begin{pmatrix} u_{A_k}^*(x_1) \\ u_{A_k}^*(x_2) \\ \vdots \\ u_{A_k}^*(x_n) \end{pmatrix} = \sum_{i=1}^n w_{A_k}(x_i) \mu_{A_k}^*(x_i) = \sum_{i=1}^n w_{A_k}(x_i) \mu_{A_k}(x_i) + \lambda \sum_{i=1}^n w_{A_k}(x_i) \pi_{A_k}(x_i). \quad (8)$$

If all the students are derived from multiple levels, then we have the following assessment formula:

$$R_M(A_k^*) = W \circ A_k^* = (w_{A_k}(x_1) \quad w_{A_k}(x_2) \quad \cdots \quad w_{A_k}(x_n)) \circ \begin{pmatrix} u_{11}(A_k^*, x_1) & u_{12}(A_k^*, x_1) & \cdots & u_{1m}(A_k^*, x_1) \\ u_{21}(A_k^*, x_2) & u_{22}(A_k^*, x_2) & \cdots & u_{2m}(A_k^*, x_2) \\ \vdots & \vdots & \cdots & \vdots \\ u_{n1}(A_k^*, x_n) & u_{n2}(A_k^*, x_n) & \cdots & u_{nm}(A_k^*, x_n) \end{pmatrix} \quad (9)$$

$$= \left(\sum_{i=1}^n w_{A_k}(x_i) u_{1i}(A_k^*, x_i) \quad \sum_{i=1}^n w_{A_k}(x_i) u_{2i}(A_k^*, x_i) \quad \cdots \quad \sum_{i=1}^n w_{A_k}(x_i) u_{mi}(A_k^*, x_i) \right).$$

Where $u_{ji}(A_k^*, x_i)$ is the fuzzy membership degree for each assessment-level.

We can evaluate the effect for the students to study the statistical software according to the standard ranking function values:

$$S \tan d(R_M(A_k^*)) = \left(\frac{\sum_{i=1}^n w_{A_k}(x_i) u_{1i}(A_k^*, x_i)}{\sum_{j=1}^m \sum_{i=1}^n w_{A_k}(x_i) u_{ji}(A_k^*, x_i)} \quad \frac{\sum_{i=1}^n w_{A_k}(x_i) u_{2i}(A_k^*, x_i)}{\sum_{j=1}^m \sum_{i=1}^n w_{A_k}(x_i) u_{ji}(A_k^*, x_i)} \quad \cdots \quad \frac{\sum_{i=1}^n w_{A_k}(x_i) u_{mi}(A_k^*, x_i)}{\sum_{j=1}^m \sum_{i=1}^n w_{A_k}(x_i) u_{ji}(A_k^*, x_i)} \right). \quad (10)$$

Where $\sum_{j=1}^m \sum_{i=1}^n w_{A_k}(x_i) u_{ji}(A_k^*, x_i) = \sum_{i=1}^n w_{A_k}(x_i) u_{1i}(A_k^*, x_i) + \sum_{i=1}^n w_{A_k}(x_i) u_{2i}(A_k^*, x_i) + \cdots + \sum_{i=1}^n w_{A_k}(x_i) u_{mi}(A_k^*, x_i).$

7. conclusion

We propose a DFS method derived from IFS, and apply it to a multiple level practical teaching assessment problem on statistical software. The DFS method not only involves the membership function and the non-

membership function, but also involves the detachment of the hesitancy function. Therefore, it is more comprehensive and flexible than the IFS method.

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